November 2023

YBMA News

The Newsletter of the Yorkshire Branch of the Mathematical Association

This newsletter marks the 32nd anniversary of the first issue which appeared in the autumn of 1991. The late 1980s were a particularly successful period for the YBMA. Membership had grown to around 80, largely the result of Tony Orton's efforts to promote the YBMA., first as its secretary and then as president, It was his idea to send out a regular newsletter to inform members about Branch matters, about the MA and about wider issues in mathematics teaching. At around the same time the number of meetings was boosted from one to two per term, so there was plenty to write about. Tony became the first editor of the newsletter and continued in this position for over 20 years until handing over to Alan Slomson in the spring of 2014.

I have looked forward to each and every issue of the YBMA News. I will strive to keep up the high standards set by my two predecessors. Regrettably, despite Tony's and Alan's efforts and the full support of the committee, YBMA membership has been falling away steadily. We have been unable to attract enough younger members to make up for inevitable losses from the older generation.

I can offer no magic remedy. Since the 1980s there have been momentous changes in secondary education - the national curriculum, school league tables and much greater pressure on classroom teachers to deliver good exam results above all else. Through NCETM, AMSP, Maths Hubs and more, the government offers maths teachers seemingly everything they might require to teach outstanding maths lessons. However, I believe there remains a need for an independent forum where teachers can share ideas and develop mathematically.

No major changes are in the pipeline for future newsletters. I expect to publish three issues per year. They will of course focus on details of forthcoming events. If you enjoyed Alan Slomson's regular feature "Mathematics in the Classroom", don't worry, that will be there too, with solutions to problems appearing in the following issue. Lastly, I do not wish to monopolise the content. Your contributions will be most welcome.

Our next Meeting

Wednesday, 6 December 2023 7pm for 7:30pm

MALL 1, School of Mathematics University of Leeds



After three years of exile in cyberspace the quiz returns as an in-person event. Wear a face mask and keep 2m away from everyone else if you must, but do come! Bring along colleagues and friends! Everyone most welcome!

Seasonal Food and Drink provided!

Prizes for All!

As in pre-pandemic years, participants will organise themselves into teams on the night.

New this year: Everyone will have the opportunity to be quiz master for one round of questions. This is not a must, but we anticipate many of you will want to take up this challenge. Some guidance is given below.

Time allowance: 5 to 10 minutes per round.

Questions should contain some mathematics or have mathematical connections. Any link with "Christmas and the New Year" can be as flimsy as you like!

You can hand out your questions on paper, write them on whiteboard/blackboard or computer-project them onto a screen. Documents in pdf format and images in jpeg/png format should present no problems.

Plain paper will be available. Participants are expected to bring their own pen/pencil and calculator.

You must also provide answers and a mark scheme. We suggest a total of 10 marks per round.

Email rgb43@gmx.com with any queries.

A Date for your Diary

Thursday, 21 March 2024 W. P. Milne lecture

Professor Nira Chamberlain OBE

President of the Mathematical Association

The lecture forms part of a KS5 Maths Day at the University of Leeds. Further details:

https://www.stem.leeds.ac.uk/mathematics/tasterday-2/

YBMA Officers 2023-24

President: Lindsey Sharp (lindseyelizab50@hotmail.com)
Secretary & Newsletter: Bill Bardelang (rgb43@gmx.com)
Treasurer: Jane Turnbull (da.turnbull@ntlworld.com)

Previous Newsletters can be found at https://www.m-a.org.uk/branches/yorkshire

Mathematics in the Classroom

Some well-known Circles, some less so

There is a unique circle passing through the vertices of a triangle, commonly called its circumcircle. Its centre is the point of concurrence of the perpendicular bisectors of the sides of the triangle. Thus the circle is readily constructed using only straight edge and compass.

The interior angle bisectors locate the centre of the incircle of the triangle, a circle touching its three sides. Extending the sides and also using the exterior angle bisectors leads us to the three excircles of the triangle. All four circles can be constructed using only straight edge and compass.



If we omit all references to a triangle, we can rephrase this as follows:

There is a unique circle passing through three given non-collinear points.

There are four circles touching each of three given non-parallel straight lines.

There are two further situations worthy of consideration:

- (i) A circle passing through two given points *A*, *B* and touching a given line *l*.
- (ii) A circle passing through a given point *A* and touching each of two given lines *l*, *m*.



Where are are the centres of such circles and can we construct them using only straight edge and compass?

Equiangular and equilateral pentagons

In the previous Newsletter we asked two questions:

Question 1

Does there exist an equiangular pentagon whose sides are all of different lengths, but such that all the ratios of the side lengths are rational numbers?

Question 2

Does there exist an equilateral pentagon whose internal angles are all different but with each of their sizes being a whole number of degrees?

Answer 1

We show first that the answer to Question 1 is "No".

Suppose that *PQRST* is a pentagon each of whose internal angles is 108° . We choose units so that *PQ* has length 1, and we suppose that the lengths of *TP*, *QR*, *ST* and *RS* are *a*, *b*, *x* and *y* respectively.

We let *KLMN* be a rectangle on which lie the vertices of the pentagon as shown.

We may find *x* and *y* in terms of *a* and *b* using the fact that KT + TN = LR + RM

and also KP + PQ + QL = NS + SM.

We leave it to the reader to check that by solving the resulting equations we obtain



$$x = \frac{1}{2} \left(\frac{(a+b)\cos 72^{\circ} + 1}{\cos 36^{\circ}} + 2(b-a)\cos 36^{\circ} \right) \text{ and } y = \frac{1}{2} \left(\frac{(a+b)\cos 72^{\circ} + 1}{\cos 36^{\circ}} + 2(a-b)\cos 36^{\circ} \right)$$

Because $\cos 36^{\circ} = \frac{\sqrt{5} + 1}{4}$ and $\cos 72^{\circ} = \frac{\sqrt{5} - 1}{4}$ it follows that $x = \frac{(a + 2b - 1) + (1 - a)\sqrt{5}}{2}$ and $y = \frac{(2a + b - 1) + (1 - b)\sqrt{5}}{2}$.

Since $\sqrt{5}$ is irrational, if *a* and *b* are rational then *x* and *y* are rational if and only if a = b = 1, in which case x = y = 1.

We have therefore shown that the only equiangular pentagons in which all the ratios of the side lengths are rational are the regular pentagons.

Answer 2

In contrast, the answer to Question 2 is "yes".

The diagram shows a pentagon formed by joining a rhombus with internal angles 100° and 80° along an edge to an equilateral triangle.

It may be seen that this produces an equilateral pentagon whose internal angles are as shown.

Many more examples of this type may easily be constructed.

